

A STUDY ON THE INVESTIGATION OF THE TRAVELING WAVE SOLUTIONS OF THE MATHEMATICAL MODELS IN PHYSICS VIA $(m + (1/G'))$ -EXPANSION METHOD

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Abstract. The Gardner equation models the propagation of dust ion acoustic waves. As a result, it has received extensive attention in the literature. For the first time, the $(m + (1/G'))$ -expansion method is used to establish novel exact wave solutions for the Gardner equation. Solutions can exhibit various types of behavior, which can be visualized using 3D and contour graphs. The results obtained demonstrate that the presented method is powerful, useful, practical, and suitable for examining the model.

Keywords: $(m + (1/G'))$ -expansion method, Gardner equation, travelling wave solutions, mathematical modelling, nonlinear partial differential equation, soliton solution.

AMS Subject Classification: 35C08.

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Received: 14 November 2023; Revised: 27 February 2024; Accepted: 8 March 2024; Published: 22 April 2024.

1 Introduction

Nonlinear partial differential equations are essential as they can be used in many field scales. For example, it helps to model nonlinear wave phenomena that occur in fields such as hydrodynamics Sohail (2021); Wang et al. (2021), plasma physics Ali et al. (2021); Wang et al (2021), biomedical Attia et al (2021); Wang (2020), vibration Wang (2021), and optics Li, Ma (2020). This way, these waves and their inherent characteristics are better understood with other references. In this study, we aim to examine the Gardner equation Wang (2022).

$$u_t + auu_x + bu^2u_x + cu_xxx = 0 \quad (1)$$

The Gardner equation belongs to the integrable, nonlinear partial differential equations category. This equation was first proposed by the famous mathematician Clifford Gardner in 1968. Because this equation can be generalized to the KdV equation, it is sometimes called the modified KdV equation. This equation is used in many application areas, such as hydrodynamics, plasma physics, and quantum field theory. Equation 1 models the propagation of dust ion acoustic waves for two temperature ions with isothermal electrons industry plasmas (Srivastava

How to cite (APA): Altan Koç, D., Gasimov, Y.S. & Bulut, H. (2024). A study on the investigation of the traveling wave solutions of the mathematical models in physics via $(m + (1/G'))$ - expansion. *Advanced Mathematical Models & Applications*, 9(1), 5-13 <https://doi.org/10.62476/amma9105>

et al., 2021; Aliev et al., 2012; Velieva & Agamalieva, 2017). The extended tanh method was used to find the traveling wave solution Allehiany (2020). The $(G'/G, 1/G)$ -expansion method and $(1/G')$ -expansion method was used in Daghan (2016). In Fu, Liu (2004), Riccati equations were used for soliton solutions. In Betchewe et al (2013), the generalized exponential rational function and Jacobi elliptical solution methods were used to examine the Gardner equation. In Wang (2022), solutions were presented using the exp-function method. Up to now, the $(m + (1/G'))$ -expansion method has not been used for the Gardner equation. In this study, the $(m + (1/G'))$ -expansion method will be used to find the analytical traveling wave solution of the Gardner equation. As it is understood from the studies, many numerical and analytical methods have been used to examine this equation. This proves the importance of this equation. Our primary motivation for writing this article is to identify new solutions to this equation. The aim of this article is to examine the valuable and unique travelling wave solutions of the Gardner equation by using the $(m + (1/G'))$ -expansion method. This method is recently developed to investigate travelling wave solutions to the NPDEs. This method is an extended version of the classic $(1/G')$ -expansion method. Specifically, when $m = 0$ solutions produced in $(1/G')$ -expansion method can be obtained. The introduction is given in Section 1. In Section 2 the $(m + (1/G'))$ -expansion method is explained. The given method is applied to obtain some exact soliton solutions in Section 3. A brief conclusion is presented in Section 4.

2 General form of the $(m+(1/G'))$ -expansion method

Consider the nonlinear partial differential equation as the following general form Durur et al (2020); Ismael et al (2022)

$$P(u, u_x, u_t, u_{xx}, u_{tt}, u_{xt}, \dots) = 0 \quad (2)$$

where is a polynomial and $u = u(x, t)$. Then, suppose the wave variables take the form $\xi = kx + \omega t$. Eq.(2) can be transformed into the nonlinear ODE,

$$U(u, u', u'', \dots) = 0 \quad (3)$$

where prime denotes the derivative with respect to ξ .

Step1

Suppose that the solution of Eq.(2) can be written as a finite power series with the form

$$u(\xi) = \sum_{i=-n}^n a_i(m + F)^i \quad (4)$$

where $a_0, a_i (i = \pm 1, \dots, \mp n)$ and m are constants. The degree of the power series is determined by considering the homogeneous balance between a nonlinear term in Eq.(2) and the highest-order derivative $F = \frac{1}{G'}$ which $G(\xi)$ satisfies

$$G'' + (\lambda + 2m\mu)G' + \mu = 0 \quad (5)$$

Step2

Substitute the solution of Eq.(3) into Eq.(4) and use Eq.(5), then collect all terms in the same order the $(m + F)^i$ to obtain the system of algebraic equations for $U, a_0, a_i (i = \mp 1, \dots, \mp n), \lambda, \mu$.

Step3

Solve the obtained system and substitute $U, a_0, a_i (i = \mp 1, \dots, \mp n)$ and the general solution of the LODE Eq.(5) in to Eq.(3) to get the exact solution of Eq. 2.

Remark 1. The solution of the LODE $G'' + (\lambda + 2m\mu)G' + \mu = 0$ is

$$G = -\frac{\mu\xi}{\lambda + 2\mu m} - \frac{A_1}{\lambda + 2\mu m} e^{-(\lambda+2\mu m)\xi} + A_2 \quad (6)$$

where A_1, A_2 are constants depending on given boundary conditions. Thus we have

$$\begin{aligned} \frac{1}{G'} &= \frac{\lambda + 2\mu m}{-\mu + (\lambda + 2\mu m)A_1(\cosh(\lambda + 2\mu m)\xi) - \sinh((\lambda + 2\mu m)\xi)} \quad (7) \\ \left(\frac{1}{G'}\right)' &= \mu\left(m + \frac{1}{G'}\right)^2 + \lambda\left(m + \frac{1}{G'}\right) - m(\lambda + \mu) \end{aligned}$$

3 Applications of the $(m + (1/G'))$ -expansion method

In this chapter, we apply of the $(m + (1/G'))$ -expansion method to obtain the exact solutions for Garder equations. To apply the $(m + (1/G'))$ -expansion method, we introduce the following transformation:

$$u(x, t) = U(\xi), \xi = kx + \omega t \quad (8)$$

Substituting Eq.(8) into Eq.(1) we have,

$$6\omega U + 3dkU^2 + 2bkU^3 + 6ck^3U'' = 0 \quad (9)$$

where $U'' = \frac{d^2U}{d\xi^2}$.

By taking the balance between u'' and u^3 , we obtain $n = 1$. When we enter the value of balance into Eq.(9), we get

$$u(\xi) = a_{-1}(m + F)^{-1} + a_0 + a_1(m + F) \quad (10)$$

Following Step 2 in the algorithm of method yields the following algebraic system,

$$\left(m + \frac{1}{G'}\right)^{-3} : 12ck^3m^2(\lambda + m\mu)^2a_{-1} + 2bka_{-1}^3 = 0,$$

$$\left(m + \frac{1}{G'}\right)^{-2} : -18ck^3m\lambda(\lambda + m\mu)a_{-1} + 3dka_{-1}^2 + 6bka_{-1}^2a_0 = 0,$$

$$\begin{aligned} \left(m + \frac{1}{G'}\right)^{-1} : & 6ck^3\lambda^2a_{-1} - 12ck^3m\lambda(\lambda + m\mu)a_{-1} + 6\omega a_{-1} + 6dka_{-1}a_0 + \\ & 6bka_{-1}a_0^2 + 6bka_{-1}^2a_1 = 0, \end{aligned}$$

$$\begin{aligned} \left(m + \frac{1}{G'}\right)^0 : & 6ck^3\lambda\mu a_{-1} + 6\omega a_0 + 3dka_0^2 + 2bka_0^3 - 6ck^3m\lambda(\lambda + m\mu)a_1 + \\ & 6dka_{-1}a_1 + 12bka_{-1}a_0a_1 = 0, \end{aligned}$$

$$\begin{aligned} \left(m + \frac{1}{G'}\right)^1 : & 6ck^3\lambda^2\mu a_1 - 12ck^3m\lambda(\lambda + m\mu)a_1 + 6\omega a_1 + 6dka_0a_1 + 6bka_0^2a_1 + \\ & 6bka_{-1}a_0a_1^2 = 0, \end{aligned}$$

$$\left(m + \frac{1}{G'}\right)^2 : 18ck^3\lambda\mu a_1 + 3dka_1^2 + 6bka_0a_1^2 = 0,$$

$$\left(m + \frac{1}{G'}\right)^3 : 12ck^3\mu^2a_1 + 2bka_1^3 = 0.$$

Solving the system of algebraic equations, we discuss the following cases;

Case 1:

$$\begin{aligned} a_{-1} &= -\frac{6m(\lambda + m\mu)\omega}{dk(\lambda + 2m\mu)}, \quad a_0 = -\frac{(6m\mu\omega)}{dk\lambda + 2dkm\mu}, \quad a_1 = 0, \quad b = \frac{d^2k}{6\omega} \\ c &= -\frac{\omega}{k^3(\lambda + 2m\mu)^2}. \end{aligned} \quad (11)$$

we can obtain the following soliton solution

$$u(x, t) = -\frac{6A1m(\lambda + 2m\mu)\omega}{dk(e^{(\lambda+2m\mu)(kx+t\omega)}(\lambda + m\mu) + A1m(\lambda + 2m\mu))} \quad (12)$$

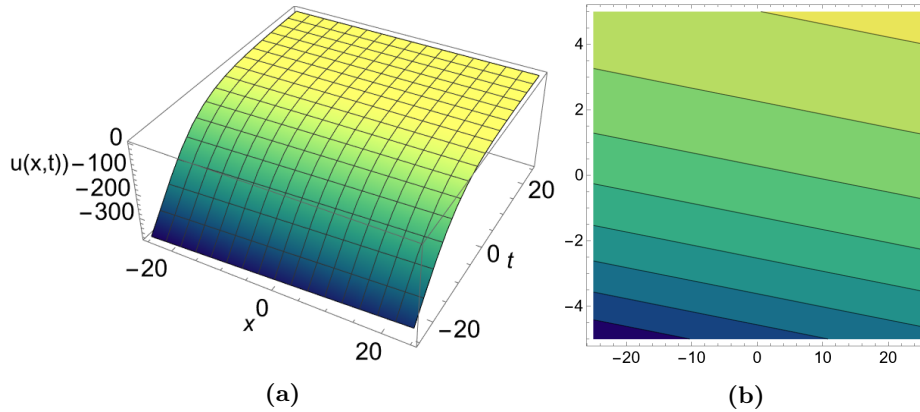


Figure 1: 3D and contour graph of traveling wave solution when $\lambda = 0.008, m = -0.1, \mu = 0.1, k = 0.2, A1 = 0.3, d = 5, \omega = 2$

Case 2:

$$\begin{aligned} a_{-1} &= -\frac{6m\lambda(\lambda + m\mu)\omega}{dk(\lambda + 2m\mu)^2}, \quad a_0 = -\frac{(6\lambda^2\omega)}{dk(\lambda + 2m\mu)^2}, \quad a_1 = -\frac{(6\lambda\mu\omega)}{dk(\lambda + 2m\mu)^2}, \quad b = \frac{d^2k(\lambda + 2m\mu)}{6\omega} \\ c &= -\frac{\omega}{k^3(\lambda + 2m\mu)^2}. \end{aligned} \quad (13)$$

we can obtain the following soliton solution

$$u(x, t) = \frac{6A1\lambda\left(\frac{1}{e^{(\lambda+2m\mu)(kx+t\omega)}\mu - A1(\lambda+2m\mu)} + \frac{m}{e^{(\lambda+2m\mu)(kx+t\omega)}(\lambda+m\mu) + A1m(\lambda+m\mu)}\right)\omega}{dk} \quad (14)$$

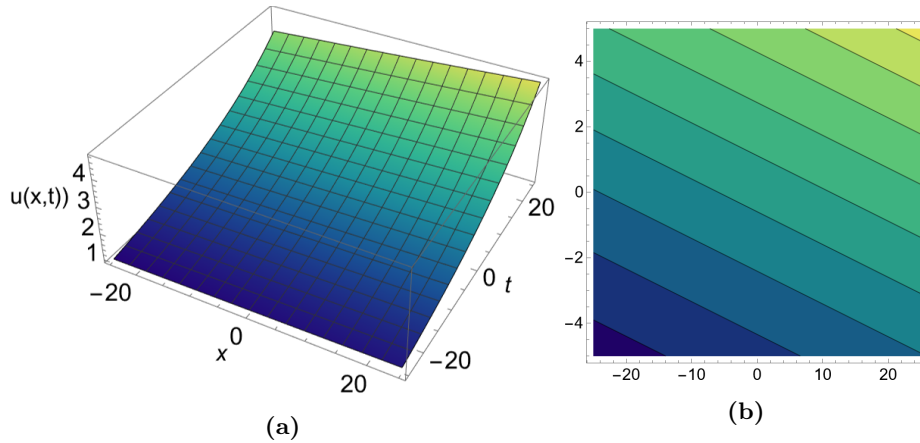


Figure 2: 3D and contour graph of traveling wave solution when $\lambda = 0.2, m = 0.1, \mu = 0.5, k = 0.02, A1 = 0.3, d = 0.5, \omega = 0.5$

Case 3:

$$\begin{aligned}
 a_{-1} &= \frac{3m(\lambda + m\mu)(k^3\lambda^2\omega - \zeta)(k^3(\lambda^2 - 32m\lambda\mu - 32m^2\mu^2)\omega)^3\zeta}{(2dk^7\lambda(\lambda + 2m\mu)^2(\lambda^2 - 32m\lambda\mu - 32m^2\mu^2)\omega)} \\
 a_0 &= \frac{12m\mu(\lambda + m\mu)k^3(\lambda^2 - 32m\lambda\mu - 32m^2\mu^2)\omega + 3\zeta}{(dk^4\lambda(\lambda + 2m\mu)^2(-\lambda^2 + 32m\lambda\mu + 32m^2\mu^2))}, c = \frac{\omega(1 - (3k^3\lambda^2\omega))}{2k^3(\lambda + 2m\mu)^2}\zeta \\
 a_1 &= \frac{3\mu(-k^3\lambda^2\omega + \zeta)(k^3(\lambda^2 - 32m\lambda\mu - 32m^2\mu^2)\omega) + 3\zeta}{(2dk^7\lambda(\lambda + 2m\mu)^2(\lambda^2 - 32m\lambda\mu - 32m^2\mu^2)\omega)} \\
 b &= \frac{d^2(-k^3(\lambda^2 - 32m\lambda\mu - 32m^2\mu^2)(\lambda^2 + 8m\lambda\mu + 8m^2\mu^2)\omega) + (\lambda^2 - 8m\lambda\mu - 8m^2\mu^2)\zeta}{1536k^2m^2\mu^2(\lambda + m\mu)^2\omega^2}.
 \end{aligned} \tag{15}$$

where $\zeta = \sqrt{k^6\lambda^2(\lambda^2 - 32m\lambda\mu - 32m^2\mu^2)\omega^2}$ we can obtain the following soliton solution

$$\begin{aligned}
 u(x, t) &= \left(\frac{1}{2dk^7(\lambda + 2m\mu)^2} 3k^3(\lambda^2 - 32m\lambda\mu - 32m^2\mu^2)\omega + 3\zeta \right) \\
 &\quad \left(\frac{8k^3m\mu(\lambda + m\mu)}{-\lambda^2 + 32m\lambda\mu + 32m^2\mu^2} + \frac{m(\lambda + m\mu)(k^3\lambda^2\omega - \zeta)}{\lambda(\lambda^2 - 32m\lambda\mu - 32m^2\mu^2)\rho(x, t)\omega} + \right. \\
 &\quad \left. \frac{\mu\rho(x, t)(-k^3\lambda^2\omega + \zeta)}{\lambda(\lambda^2 - 32m\lambda\mu - 32m^2\mu^2)\omega} \right)
 \end{aligned} \tag{16}$$

where $\rho(x, t) = m + \frac{1}{A1e^{-(\lambda+2m\mu)(kx+t\omega)} - \frac{\mu}{\lambda+2m\mu}}$.

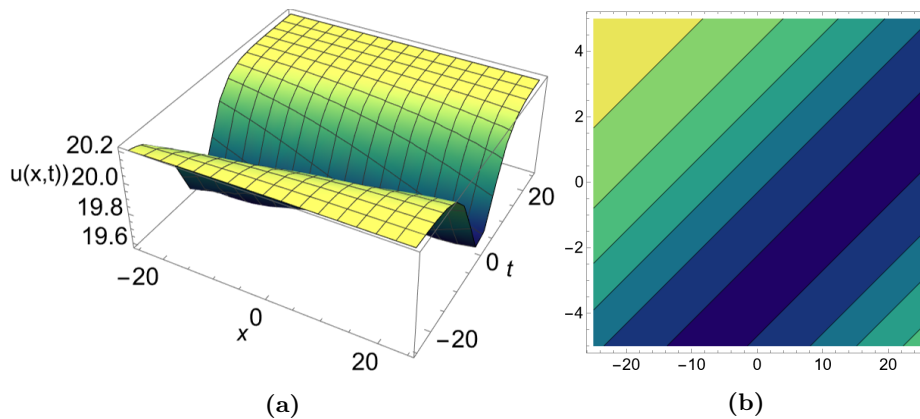


Figure 3: 3D and contour graph of traveling wave solution when $\lambda = -1, m = 0.5, \mu = 0.7, k = -0.2, A1 = 9, d = 1, \omega = 1$

Case 4:

$$a_{-1} = -\frac{6c^{1/3}m\lambda(\lambda+m\mu)\omega^{2/3}}{d(\lambda+2m\mu)^{4/3}}, \quad a_0 = \frac{(6^{1/3}\lambda^2\omega^{2/3})}{d(\lambda+2m\mu)^{4/3}}, \quad a_1 = \frac{(6c^{1/3}\lambda\mu\omega^{2/3})}{d(\lambda+2m\mu)^{4/3}}, \quad b = -\frac{d^2(\lambda+2m\mu)^{4/3}}{6c^{1/3}\lambda^2\omega^{2/3}}$$

$$k = -\frac{\omega^{1/3}}{c^{1/3}(\lambda+2m\mu)^{2/3}}.$$
(17)

we can obtain the following soliton solution

$$u(x, t) = \frac{6A1c^{1/3}e^{\kappa(x)+t(\lambda+2m\mu)\omega} \lambda(\lambda+2m\mu)^{5/3}\omega^{2/3}}{d(-e^{t(\lambda+2m\mu)\omega} \mu + A1e^{\kappa(x)}(\lambda+2m\mu))e^{t(\lambda+2m\mu)\omega(\lambda+m\mu)+A1e^{\kappa(x)}m(\lambda+2m\mu)}} \quad (18)$$

where $\kappa(x) = \frac{x(\lambda+2m\mu)^{1/3}\omega^{1/3}}{c^{1/3}}$

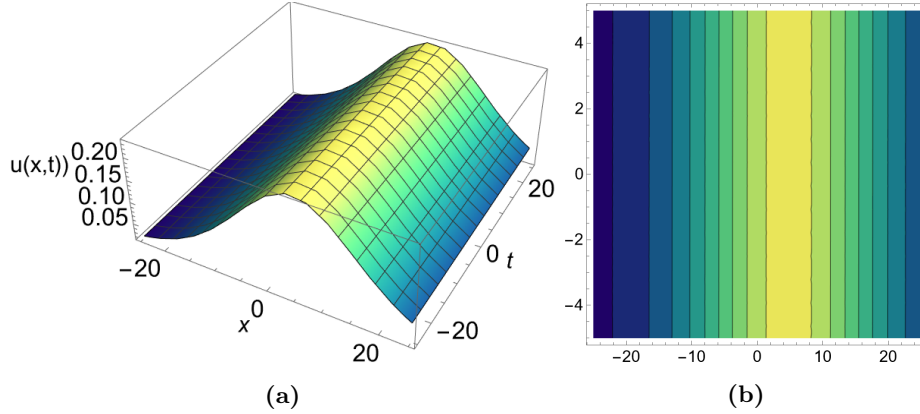


Figure 4: 3D and contour graph of traveling wave solution when $\lambda = 0.02, m = 0.5, \mu = -0.01, k = 0.2, A1 = 0.9, d = 0.05, \omega = 0.05$

Case 5:

$$a_{-1} = -\frac{6m(\lambda+m\mu)\sqrt{-\frac{cd^2\lambda^2\omega^{4/3}}{(-c(\lambda+2m\mu)^2)^{1/3}}}}{d^2\lambda}, \quad a_1 = 0, \quad k = \frac{\omega^{1/3}}{(-c(\lambda+2m\mu)^2)^{1/3}}$$

$$a_0 = \frac{3(-d(-c(\lambda+2m\mu)^2)^{1/3}\omega^{2/3} + \sqrt{-\frac{cd^2\lambda^2\omega^{4/3}}{(-c(\lambda+2m\mu)^2)^{1/3}}})}{d^2}, \quad b = \frac{d^2}{6(-c(\lambda+2m\mu)^2)^{1/3}\omega^{2/3}}$$
(19)

we can obtain the following soliton solution

$$u(x, t) = \frac{6m(\lambda+m\mu)\sqrt{-\frac{cd^2\lambda^2\omega^{4/3}}{\nu}}}{d^2\lambda\left(m + \frac{1}{A1e^{(-\lambda-2m\mu)(\frac{x\omega^{1/3}}{\nu}+t\omega)} - \frac{\mu}{\lambda+2m\mu}}\right)} - \frac{3(d\nu)\omega^{2/3} + \sqrt{-\frac{cd^2\lambda^2\omega^{4/3}}{\nu}}}{d^2} \quad (20)$$

where $\nu = (-c(\lambda+2m\mu)^2)^{1/3}$

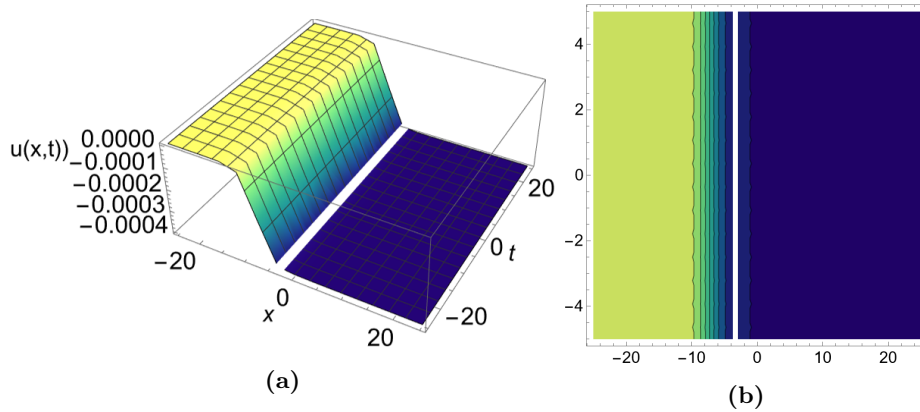


Figure 5: 3D and contour graph of traveling wave solution when $\lambda = 0.01, m = 0.005, \mu = 0.01, k = -0.000002, A1 = 0.5, d = 0.01, \omega = 0.0005$

4 Result and Discussion

In this paper, the travelling wave solutions for the Gardner equation have been studied. Exact solutions for a suggested equation are constructed by using $(m + (1/G'))$ -expansion method. Solutions are drawn to understand the physical phenomena of the Gardner equation. We have presented three-dimensional and contour graphs of the solutions to improve understanding of the characteristics of the traveling wave solutions. Figure 1 represents the 3D and contour graph of traveling wave solution in Case 1 which have been plotted for the values $\lambda = 0.2, m = 0.1, \mu = 1/2, k = 0.02, A1 = 0.3, d = 0.5, \omega = 0.5$. This type of solution is a traveling wave solution with real structure. The three dimensional and contour graphs of soliton solution are plotted in Figure 2 for the values $\lambda = 0.008, m = -0.1, \mu = 0.1, k = 0.2, A1 = 0.3, d = 5, \omega = 2$. In Case 3, a different soliton solution has been obtained with the help of the selection of coefficients. The three dimensional and contour graph of this solution are plotted in Figure 3 for the values $\lambda = -1, m = 0.5, \mu = 0.7, k = -0.2, A1 = 9, d = 1, \omega = 1$. In Case 4 the real soliton solution has been obtained. Its three dimensional and contour graph are plotted in Figure 4 for the values $\lambda = 0.02, m = 0.5, \mu = -0.01, k = 0.2, A1 = 0.9, d = 0.05, \omega = 0.05$. The solution in Case 5, in a real form. Its graph has been plotted in Figure 5 for the $\lambda = 0.01, m = 0.005, \mu = 0.01, k = -0.000002, A1 = 0.5, d = 0.01, \omega = 0.0005$. The given results could be helpful in explaining the physical meaning of different nonlinear models.

5 Conclusion

This article examines the application of the $(m + (1/G'))$ -expansion method to the Gardner equation to generate the traveling wave solutions. This approach is an effective technique that can be applied to nonlinear-type equations. Fully propagated wave solutions are obtained as bright solitary, bright-dark solitary, and periodic wave solutions. For more understanding of the physical phenomena of the obtained solutions, they are plotted in 3D and contour graphics. The results in this article show that the $(m + (1/G'))$ -expansion method is concise, efficient, and can offer different forms of traveling wave solutions. It shows that it can be used better to understand the complex physical phenomena in the field. We note that all the solutions obtained confirm the given equation when directly substituted.

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